

부록 A

"살아 있는" 수학책

이 책의 대부분은 지역 도서관이나 학술자료 교환을 통해 구할 수 있습니다. 많은 오래된 서적이 절판되었지만 여전히 훌륭합니다. 더 많은 책(링크 포함)은 살아 있는 책 (denisegaskins.com/living-math-books) 블로그 페이지에서 확인할 수 있습니다.

그리고 내가 놓친 위대한 책을 발견하면, 편지를 써주세요. 즉시 그것을 학술자료 교환 목록에 추가할 것입니다.

학부모 및 교사용

Adding It Up: Helping Children Learn Mathematics by Jeremy Kilpatrick, et al.

Children Doing Mathematics by Terezinha Nunes and Peter Bryant

Children's Mathematics: Cognitively Guided Instruction by Thomas P. Carpenter, et al.

Creative Problem Solving in School Mathematics by George Lenchner

Dr. Wright's Kitchen Table Math by Chris Wright

Elementary Mathematics for Teachers and *Elementary Geometry for Teachers* by Thomas H. Parker and Scott J. Baldridge

Extending Children's Mathematics: Fractions & Decimals by Susan B. Empson and Linda Levi

Family Math by Stenmark, et al.

First Grade Diary by Lore Rasmussen

Games for Math by Peggy Kaye

Good Questions for Math Teaching: Why Ask Them and What to Ask, Grades K-6 by Peter Sullivan and Pat Lilburn, *Grades 5-8* by Nancy Canavan Anderson and Lainie Schuster

How to Homeschool Math—Even If You Hate Fractions!! by Robin Padron

How to Solve It: A New Aspect of Mathematical Method by George Polya

Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States by Liping Ma

Math by Kids! A Collection of Word Problems Written by Kids for Kids of All Ages edited by Susan Richman

Math from Three to Seven: The Story of a Mathematical Circle for Preschoolers by Alexander Zvonkin, *Mathematical Circle Diaries, Year 1: Complete Curriculum for Grades 5 to 7* by Anna Burago, and other books in the MSRI Mathematical Circles Library series

Math You Can Play series by Denise Gaskins

Mathematical Activities: A Resource Book for Teachers and other books by Brian Bolt

A Mathematician's Lament: How School Cheats Us out of Our Most Fascinating and Imaginative Art Form by Paul Lockhart

Mindset: The New Psychology of Success by Carol Dweck

Moebius Noodles: Adventurous Math for the Playground Crowd by Yelena McManaman and Maria Droujkova

The Myth of Ability: Nurturing Mathematical Talent in Every Child by John Mighton

Nix the Tricks: A guide to avoiding shortcuts that cut out math concept development by Tina Cardone

Number Sense Routines: Building Numerical Literacy Every Day in Grades K–3 by Jessica F. Shumway

Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K–5 by Sherry Parrish

Old Dogs, New Math: Homework Help for Puzzled Parents by Mike Askew and Rob Eastaway

Playing with Math: Stories from Math Circles, Homeschoolers, and Passionate Teachers edited by Sue VanHattum

Talking Math with Your Kids and other books by Christopher Danielson

The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom by James W. Stigler, James Hiebert

Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction by John A. Van de Walle et al.

Vision in Elementary Mathematics and other books by W. W. Sawyer

What's Math Got to Do with It? How Teachers and Parents Can Transform Mathematics Learning and Inspire Success by Jo Boaler

취학 전 아동과 초등 저학년용

Anything by Greg Tang, Tana Hoban, or Mitsumasa Anno

12 Ways to Get to 11 by Eve Merriam

A Fly on the Ceiling by Julie Glass

Apple Fractions by Jerry Pallotta

Ben Franklin and the Magic Squares by Frank Murphy

Blockhead: The Life of Fibonacci by Joseph D'Agnese

The Boy Who Loved Math: The Improbable Life of Paul Erdos by Deborah Heiligman
Camp Logic by Mark Saul and Sian Zelbo
Count On Your Fingers African Style by Claudia Zaslavsky
Dear Benjamin Banneker by Andrea Davis Pinkne
Each Orange Had 8 Slices by Paul Giganti
Fraction Fun by David A. Adler
Full House by Dayle Ann Dodds
Grandfather Tang's Story by Ann Tompert
Growing Patterns by Sarah C. Campbell
How Much Is a Million? and other books by David Schwartz
The Librarian Who Measured the Earth by Kathryn Lasky
MathStart series by Stuart J. Murphy
Multiplying Menace and *The Multiplying Menace Divides* by Pam Calvert
Of Numbers and Stars: The Story of Hypatia by D. Anne Love
One Hundred Hungry Ants and *A Remainder of One* by Elinor J. Pinczes
Rabbits, Rabbits Everywhere: A Fibonacci Tale by Ann McCallum
Sir Cumference series by Cindy Neuschwander
Spaghetti and Meatballs for All! and *The Greedy Triangle* by Marilyn Burns
Tangramables by Judi Martschinke
Ten, Nine, Eight by Molly Bang
Three Pigs, One Wolf, Seven Magic Shapes by Grace Maccarone
 The Time-Life *I Love Math* series (various authors)
What's Your Angle, Pythagoras? A Math Adventure by Julie Ellis
Which One Doesn't Belong? by Christopher Danielson (2016)
You Can Count on Monsters by Richard Evan Schwartz
Young Math Books series by Crowell (various authors)

초등 고학년과 중학생용

Many of the above, plus:

The Adventures of Penrose the Mathematical Cat and other books by Theoni Pappas
The Amazing Mathematical Amusement Arcade and other books by Brian Bolt
Archimedes and the Door of Science by Jeanne Bendick
Can You Count in Greek?: Exploring Ancient Number Systems by Judy Leimbach
Carry On, Mr. Bowditch by Jean Lee Latham
The Cat in Numberland by Ivar Ekeland
Challenge Math and other books by Ed Zaccaro

Classic Brain Teasers by Martin Gardner
Compass Drawings by Linda Nelson Harst and Margaret Sayre Wiederhold
Competition Math for Middle School by J. Batterson
Cool Math! by Christy Maganzini
Curve Stitching: Art of Sewing Beautiful Mathematical Patterns by Jon Millington
Do You Wanna Bet? by Jean Cushman
Fibonacci Fun by Trudi Hammel Garland
G is for Googol by David Schwarz
Geometrical Design, Visual Illusions and other coloring books from Dover
Go Figure! A Totally Cool Book about Numbers by Johnny Ball
The History of Counting by Denise Schmandt-Besserat
Introduction to Tessellations by Jill Britton
The Man Who Counted: A Collection of Mathematical Adventures by Malba Tahan
Material World by Peter Menzel
Math by Kids! edited by Susan Richman
Math Curse by Jon Scieszka
Math Dictionary for Kids: The Essential Guide to Math Terms, Strategies, and Tables by
Theresa Fitzgerald
Math Doesn't Suck and *Kiss My Math* by Danica McKellar
Math for Smarty Pants or *The 'I Hate Math' Book* by Marilyn Burns
Math Games and Activities from Around the World and other books by Claudia
Zaslavsky
Math Olympiad Contest Problems by George Lenchner
Math on Call: A Mathematics Handbook by Andrew Kaplan et al.
Math Without Words by James Tanton
Mathematicians Are People, Too (and *Volume 2*) by Luetta and Wilbert Reimer
Murderous Maths series by Kjartan Poskitt
The Number Devil: A Mathematical Adventure by Hans Magnus Enzensberger
The Phantom Tollbooth by Norton Juster
Puzzlegams and *More Puzzlegams* compiled by Pentagram
Sam Loyd's Book of Tangrams by Sam Loyd
The Secret Life of Math and others by Ann McCallum
Sideways Arithmetic from Wayside School and *More Sideways Arithmetic* by Louis
Sachar
String, Straight-edge & Shadow: The Story of Geometry by Julia E. Diggins
This Is Not a Math Book by Anna Weltman
The Wonderful World of Mathematics by Lancelot Hogben

청소년과 성인용

Many of the above, plus:

All the Math You'll Ever Need by Steve Slavin

The Art and Craft of Problem Solving by Paul Zeitz

Calculus Made Easy by Silvanus P. Thompson and Martin Gardner

The Cartoon Guide to Statistics and others by Larry Gonick

The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems and other books by Martin Gardner

The Complete How to Figure It by Darrell Huff

Crocheting Adventures with Hyperbolic Planes by Daina Taimina

Flatland by Edwin A. Abbott

Games with Pencil and Paper by Eric Solomon

Geometry, Relativity and the Fourth Dimension by Rudolf v. B. Rucker

Hot X: Algebra Exposed and *Girls Get Curves* by Danica McKellar

How to Lie with Statistics by Darrell Huff

How Not to Be Wrong: The Power of Mathematical Thinking by Jordan Ellenberg

How to Solve It by George Polya

Innumeracy: Mathematical Illiteracy and Its Consequences and *A Mathematician Reads the Newspaper* by John Allen Paulos

Invent Your Own Computer Games with Python by Al Sweigart, also available for free download at InventWithPython.com

The Joy of Mathematics and other books by Theoni Pappas

The Joy of x : A Guided Tour of Math, from One to Infinity and other books by Steven Strogatz

The Language of Mathematics: Making the Invisible Visible and other books by Keith Devlin

Lateral Thinking Puzzlers by Paul Sloane

Magical Mathematics: The Mathematical Ideas That Animate Great Magic Tricks by Persi Diaconis and Ron Graham

The Manga Guide to Calculus by Hiroyuki Kojima and Shin Togami

Math Girls by Hiroshi Yuki

Math Jokes 4 Mathy Folks by G. Patrick Vennebush

Mathematical Circles: Russian Experience by Dmitri Fomin et al.

Mathematician's Delight by W. Sawyer

Mathematics: A Human Endeavor by Harold R. Jacobs

Mathematics for the Nonmathematician by Morris Kline

Measurement by Paul Lockhart

The Moscow Puzzles: 359 Mathematical Recreations by Boris A. Kordemsky
Naked Statistics: Stripping the Dread from the Data by Charles Wheelan
The Number Mysteries: A Mathematical Odyssey through Everyday Life and other books
 by Marcus du Sautoy
A Passion for Mathematics and other books by Clifford A. Pickover
The Princeton Companion to Mathematics by Timothy Gowers et al.
Prisoner's Dilemma by William Poundstone
Problem Solving Through Recreational Mathematics by Bonnie Averbach and Orin
 Chein
Prof. E. McSquared's Calculus Primer: Expanded Intergalactic Version by Howard
 Swann and John Johnson
Professor Stewart's Hoard of Mathematical Treasures and other books by Ian Stewart
Relativity in Illustrations by Jacob T. Schwarz
Ruler and Compass: Practical Geometric Constructions by Andrew Sutton
The Signal and the Noise: Why So Many Predictions Fail—but Some Don't by Nate
 Silver
Solve This: Math Activities for Students and Clubs by James Tanton
*Standard Deviations: Flawed Assumptions, Tortured Data, and Other Ways to Lie with
 Statistics* by Gary Smith
Statistics Done Wrong: The Woefully Complete Guide by Alex Reinhart
Taxicab Geometry: An Adventure in Non-Euclidean Geometry by Eugene F. Krause
Thinking Mathematically by John Mason
The Visual Guide to Extra Dimensions by Chris McMullen
What Is the Name of This Book? and other collections of logic puzzles by Raymond
 Smullyan

수학 역사와 전기문

Ada's Algorithm: How Lord Byron's Daughter Ada Lovelace Launched the Digital Age by
 James Essinger
Adventures of a Mathematician by S. M. Ulam
Agnesi to Zeno: Over 100 Vignettes from the History of Math by Sanderson Smith
A Beautiful Mind by Sylvia Nasar
The Code Book: The Science of Secrecy from Ancient Egypt to Quantum Cryptography by
 Simon Singh
Count Like an Egyptian: A Hands-on Introduction to Ancient Mathematics by David
 Reimer
Descartes: A Biography by Desmond M. Clarke
"e": The Story of a Number by Eli Maor

Einstein: His Life and Universe by Walter Isaacson
Euler: The Master of Us All by William Dunham
Euler's Gem: The Polyhedron Formula and the Birth of Topology by David S. Richeson
Famous Problems and Their Mathematicians by Art Johnson
Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem by Simon Singh
Gödel, Escher, Bach: An Eternal Golden Braid by Douglas R. Hofstadter
Hilbert by Constance Reid
Historical Connections in Mathematics by Luetta and Wilbert Reimer
The Honors Class: Hilbert's Problems and Their Solvers by Benjamin Yandell
I Want to Be a Mathematician: An Autobiography in Three Parts by Paul R. Halmos
In Code: A Mathematical Journey by Sarah Flannery
John Von Neumann: The Scientific Genius Who Pioneered the Modern Computer, Game Theory, Nuclear Deterrence, and Much More by Norman MacRae
Journey through Genius and other books by William Dunham
King of Infinite Space: Donald Coxeter, the Man Who Saved Geometry by Siobhan Roberts
Leibniz: An Intellectual Biography by Maria Rosa Antognazza
The Man Who Knew Infinity: A Life of the Genius Ramanujan by Robert Kanigel
The Math Book: From Pythagoras to the 57th Dimension, 250 Milestones in the History of Mathematics by Clifford A. Pickover
Math Equals by Teri Perl
Math through Children's Literature: Making the NCTM Standards Come Alive by Kathryn L. Braddon et al.
Math through the Ages: A Gentle History for Teachers and Others by William P. Berlinghoff
Mathematics: From the Birth of Numbers by Jan Gullberg
Mathematics Frontiers and other books in the *Pioneers in Mathematics* series by Michael J. Bradley
The Mathmen by Leon Terry
Men of Mathematics by Eric Temple Bell
The Music of the Primes: Searching to Solve the Greatest Mystery in Mathematics by Marcus du Sautoy
My Brain Is Open: The Mathematical Journeys of Paul Erdos by Bruce Schechter
Never at Rest: A Biography of Isaac Newton by Richard S. Westfall
Of Men and Numbers: The Story of the Great Mathematicians by Jane Muir
The Poincaré Conjecture: In Search of the Shape of the Universe by Donal O'Shea
Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics by John Derbyshire

Remarkable Mathematicians: From Euler to von Neumann by Ioan James

Unknown Quantity: A Real and Imaginary History of Algebra by John Derbyshire

Women and Numbers by Teri Perl and Analee Nunan

Women in Mathematics by Lynn M. Osen

The World of Mathematics: A Four-Volume Set by James R. Newman

...And there are plenty more where these came from, enough to keep you exploring mathematics for a lifetime.

부록 B

인터넷상의 수학 자료

인터넷은 광범위한 수학 웹사이트로 가득합니다. 다음은 내가 좋아하는 학교 수학 및 재미있는 것들 중 일부입니다. 이 링크들은 내 웹사이트(denisegaskins.com/internet-math-resources)에서도 이용할 수 있습니다.

여기에 소개하는 모든 웹사이트 링크는 2015년 12월에 확인하였습니다. 블로그 목록을 최신 상태로 유지하기 위해 최선을 다했지만 인터넷 환경은 언제 바뀔지 모릅니다. 만약 웹사이트가 사라졌다면 저자명이나 글 제목으로 검색해 찾을 수 있습니다. 웹기록보관소(archive.org/web/web.php)에 웹사이트 주소를 입력해도 찾을 수 있습니다.

학부모 및 교사용

ADDING IT UP, HELPING CHILDREN LEARN MATHEMATICS: An overview of research about elementary school arithmetic and how to teach it, free for downloading or reading online.

nap.edu/openbook.php?record_id=9822&page=71

AND 'RITHMETIC: The Sudbury Valley School approach to math education, by Daniel Greenberg.

scribd.com/doc/14389275/And-Rithmetic-by-Daniel-Greenberg

ART OF PROBLEM SOLVING MATH ARTICLES: A variety of topics about teaching and learning math.

artofproblemsolving.com/articles

CLOTHESLINE MATH: Harness the power of the number line to build understanding from early elementary school to high school.

clotheslinemath.com

DENISEGASKINS.COM: My "Let's Play Math!" blog of games, teaching tips, and resource pages. For instance, check out my Word Problems from Literature series.

denisegaskins.com

denisegaskins.com/2010/04/26/word-problems-from-literature

EDUCATION UNBOXED: Videos by homeschooling mom Rosie showing how to play with elementary math using Cuisenaire rods and other hands-on tools.

educationunboxed.com

ELEMENTS OF MATH: Steven Strogatz's blog post series from *The New York Times*. His

안전성을 유지하기 위해

훌륭한 대화형 수학 웹 애플리케이션들은 대개 자바나 어도비 플래시가 필요하다. 그러나 두 프로그램 모두 해커들이 다른 사람의 컴퓨터에 침입하거나 다른 못된 짓을 저지를 때 사용할 위험이 있다. 둘 중 어느 것을 사용하든 반드시 최신 버전으로 사용하고 컴퓨터 보안 환경을 업데이트하는 것을 잊지 말자. 자바 기반 애플리케이션을 사용하려면 해당 웹사이트를 자바스크립트 허용 사이트 목록에 추가해야 할 것이다.

java.com

java.com/en/download/faq/exception_sitelist.xml

adobe.com/products/flashplayer.html

“Me, Myself and Math” series is also worth reading.

topics.nytimes.com/top/opinion/series/steven_strogatz_on_the_elements_of_math/index.html

opinionator.blogs.nytimes.com/category/me-myself-and-math/

EVER WONDER WHAT THEY’D NOTICE? (IF ONLY SOMEONE WOULD ASK): Annie Fetter’s classic presentation on talking with kids about math.

youtu.be/a-Fth6sOaRA

GLOBAL STRATEGY STAGE ASSESSMENT (GLOSS): A one-on-one test of mathematical understanding. You read a question, give your child time to think, and then compare what he or she says to the range of possible responses. Be sure to read the Additional Information about Gloss (PDF) before interviewing your student.

nzmaths.co.nz/gloss-forms?parent_nod

nzmaths.co.nz/sites/default/files/Numeracy/Gloss/Gloss_Additional_Information.pdf

THE HARMFUL EFFECTS OF “CARRYING” AND “BORROWING” (PDF): Teaching abstract rules too soon can damage children’s intuition about numbers.

sites.google.com/site/constancekamii/articles-available-for-printing/The_Harmful_Effects_of_Carrying_and_Borrowing_%282009%29.pdf

HELPING A STRUGGLING MATH STUDENT: A 14-part series at Angelicscalliwags blog, full of activities and encouragement. Scroll to the bottom to read the posts in the order they were published.

angelicscalliwags.com/category/helping-a-struggling-maths-student

HOW TO THINK LIKE A SCHOOL MATH GENIUS: James Tanton’s series of videos about five key principles for mathematical thinking.

jamestanton.com/?p=1097

KENKEN FOR TEACHERS: A fantastic way to practice arithmetic.

kenkenpuzzle.com/teachers/classroom

LIVING MATH: Julie Brennan's amazing website features the most extensive lists of living math books anywhere, plus articles about math, book and resource reviews, and lesson plans. Brennan also moderates the Yahoo group, Living Math Forum.

livingmath.net

groups.yahoo.com/group/LivingMathForum

MATH BY KIDS: A 78-page workbook of original math problems (including solutions) created by homeschooled students aged four to seventeen, edited by Susan Richman. If this store link stops working, go to the Pennsylvania Homeschoolers website and click around until you find the book.

pahomeschoolers.c9.ixsecure.com/oscommerce-2.3.3/catalog/product_info.php?products_id=42

pahomeschoolers.com

MATH JOURNALS BOOST REAL LEARNING (PDF): An article by Marilyn Burns. Math journals "help students stretch their thinking and make sense of problems," and they can help teachers evaluate student progress.

coach4math.com/wp-content/uploads/2010/03/Math-Journals-Boost-Real-Learning-Article.pdf

MATH REASONING INVENTORY: Find out how much your elementary or middle school students understand about math. You do not have to sign up for an account to access the resources. Before doing the oral assessment testing, be sure to read the Reasoning Strategy PDF Files that outline what the assessment is looking for and watch the videos to see what sort of answers to expect.

mathreasoningininventory.com

mathreasoningininventory.com/Home/Resources

MATH TALKS: Fawn Nguyen's collection of questions that spark thinking about math, with sample student answers and tips for the teacher.

mathtalks.net

MODERN MATH FOR ELEMENTARY SCHOOLERS (PDF): Downloadable Creative Commons (CC BY-NC-SA 3.0) book by Oleg Gleizer with more than two hundred puzzles about numbers, geometry, infinity, and more.

old.naturalmath.com/DeltaStreamMedia/OlegGleizerModernMathematics_12_2011.pdf

MOEBIUS NOODLES: "Adventurous math for the playground crowd." Plenty of ideas for sharing rich math experiences with your children.

moebiusnoodles.com

NIX THE TRICKS: Tina Cardone's free guide explains how mnemonic tricks and shortcuts hinder student understanding of math. Learn which phrases to avoid and what to use in their place.

nixthetricks.com

PROBLEM SOLVING STRATEGIES: Finlay McQuade's tips for teaching your students to solve math problems.

pred.boun.edu/tr/ps

RELATIONAL UNDERSTANDING AND INSTRUMENTAL UNDERSTANDING (PDF): The original article by Richard Skemp on the two ways of understanding mathematics. math.coe.uga.edu/olive/EMAT3500f08/instrumental-relational.pdf

SNAP—SCAFFOLDING FOR NUMERICAL SYNAPSES: Montessori-influenced activities by Sheryl Morris. Help preschool children find numbers, their related patterns, and geometric shapes in the world all around them. snap-scaffoldingfornumericalsynapses.com

STANDARDS FOR MATHEMATICAL PRACTICE: The best part of the Common Core Math reform, a summary of what it means to think mathematically from kindergarten to high school. corestandards.org/Math/Practice

TALKING MATH WITH YOUR KIDS: Christopher Danielson helps parents support their children's mathematical development. talkingmathwithkids.com

TALKING STICK MATH CIRCLE BLOG: Inspiring stories about children grappling with math concepts, by Rodi Steinig. talkingsticklearningcenter.org/category/math-circle-blog

THE TEACHING OF ARITHMETIC: The Story of an Experiment: In 1929, American school superintendent Louis P. Benezet delayed arithmetic to help students build a foundation in reasoning. wol.ra.phy.cam.ac.uk/sanjoy/benezet/three.html

ULTIMATE LIST OF PRINTABLE MATH MANIPULATIVES & GAMES: A treasure list from Jimmie Lanley, one of my favorite homeschooling bloggers. jimmiescollage.com/2011/04/ultimate-list-of-printable-math-manipulatives-games

UNSCHOOLERS AND MATHEMATICS: Sandra Dodd's collection of stories about children learning math naturally, without being pushed to use textbooks or to drill math facts. Inspiring. sandraddodd.com/math

THE WORLD OF MATHEMATICAL REALITY: Catch a vision of mathematical beauty in this video by Paul Lockhart. See also his essay "A Mathematician's Lament" (PDF). youtu.be/V1gT2f3Fe44
maa.org/external_archive/devlin/LockhartsLament.pdf

WRITING IN MATHEMATICS: Terry Kawas's tips to help students reflect on their learning, deepen their understanding, and make important connections to real-life applications. mathwire.com/writing/writing1.html

YELENA'S HUNDRED CHART POSTER: Printable hundred chart poster and game cards from Moebius Noodles blogger Yelena McManaman. moebiusnoodles.com/2013/01/the-hundred-chart-and-game-cards

YOU CUBED: Jo Boaler's website for research-based ways to teach math that can help

discouraged students feel empowered to learn. Also try her free online course for students and their parents: How to Learn Math.

youcubed.org

youcubed.org/how-to-learn-math-for-students

수학 탐험

ALICE: A downloadable programming system that allows students to learn by creating animated movies and simple video games.

alice.org/index.php

CODE.ORG: Game-like computer programming tutorials for beginners and beyond.

code.org/learn

CODECADEMY: Interactive lessons in a variety of programming languages, along with project ideas and forums for asking questions.

codecademy.com

CS UNPLUGGED: Learning activities that teach computer science through games and puzzles using cards, string, crayons and lots of running around.

csunplugged.org

CUT THE KNOT INTERACTIVE: Alexander Bogomolny's "Mathematics Miscellany and Puzzles," one of my all-time favorite sites.

cut-the-knot.org

DAILY TREASURE: Solve the logic puzzle to find the hidden gold.

4chests.blogspot.com

DON COHEN'S MAP OF CALCULUS FOR YOUNG PEOPLE: Hands-on activities featuring advanced ideas, for students of any age. Check out my introductory blog post Infinite Cake: Don Cohen's Infinite Series for Kids.

web.archive.org/web/20160325044804/http://mathman.biz/html/map.html

denisegaskins.com/2015/07/02/infinite-cake-don-cohens-infinite-series-for-kids

DOODLING IN MATH: Recreational mathematics and inspirational videos by math-emu musician Vi Hart. For newer videos, see her blog ViHart.com.

khanacademy.org/math/recreational-math/vi-hart

ESTIMATION 180: Andrew Stadel's site for "Building number sense one day at a time." How close can you guess?

estimation180.com

FAMOUS PROBLEMS IN THE HISTORY OF MATHEMATICS: This site includes problems, paradoxes, and proofs that have inspired mathematicians through the ages, plus links for further exploration.

mathforum.org/isaac/mathhist.html

FUN MATHEMATICS LESSONS BY CYNTHIA LANIUS: A variety of topics and investigations.

math.rice.edu/~lanius/Lessons

GEOMETRY LESSONS IN THE WALDORF SCHOOL: Freehand Form Drawing and Basic

Geometric Construction: (Includes link to free PDF download) The book says “Grades 4 and 5,” but Waldorf-style geometry doodling is fun for all ages.
waldorflibrary.org/books/3/view_bl/113/form-drawing/120/geometry-lessons-in-the-waldorf-school-ebook

GOLDEN SALES PITCH: “There is little evidence to suggest that the golden ratio has any special aesthetic appeal ... When a myth is repeated over and over, it begins to sound like truth.”
sciencenews.org/view/generic/id/8660/title/A_Golden_Sales_Pitch

HISTORY OF MATHEMATICAL GAMES AND RECREATIONS: “The whole history of mathematics is interwoven with mathematical games which have led to the study of many areas of mathematics.”
www-groups.dcs.st-and.ac.uk/~history/HistTopics/Mathematical_games.html

HOTEL INFINITY: Tova Brown’s growing collection of videos that explore advanced math concepts through story-telling.
hotel-infinity.com

ISLAMIC ART AND GEOMETRIC DESIGN (PDF): Lesson plans from the Metropolitan Museum.
metmuseum.org/~media/Files/Learn/For_Educators/Publications_for_Educators/Islamic_Art_and_Geometric_Design.pdf

JILL BRITTON’S HOME PAGE: A wealth of links and resources for playing with topology, symmetry, tessellations, and polyhedra.
britton.disted.camosun.bc.ca/home.htm

MATH HOMBRE GAMES: Links to math games on GVSU math professor John Golden’s blog, games on other people’s blogs, and more games all over the Internet.
bit.ly/mhGames

MATH IS FUN: A mathematical smorgasbord of lessons, definitions, puzzles, and games.
mathsisfun.com/index.htm

MATH MUNCH: A weekly digest of delicious math activities, projects, artwork, and games from around the Internet.
mathmunch.org

MATH PICKLE: Gordon Hamilton offers challenging printable games/puzzles for K–12 students. Can your kids solve the \$1,000,000 problems?
mathpickle.com

NATIONAL LIBRARY OF VIRTUAL MANIPULATIVES: A treasure-chest of virtual hands-on math. Includes links to material for all ages and topics, pre-K through 12th grade.
nlvm.usu.edu/en/nav/vlibrary.html

NRICH.MATHS.ORG: A wonderful source of math puzzles and activities for all ages, with a theme that changes each month. Hints available, and solutions for past problems.
nrich.maths.org/public/index.php

ORIGAMI AND MATH: David Eppstein's Geometry Junkyard links to a slew of origami articles and projects.

ics.uci.edu/~eppstein/junkyard/origami.html

ORIGAMI INSTRUCTIONS: You can make a variety of polyhedra from Sonobe modules.

See also Wikipedia: Sonobe.

origami-instructions.com/modular-sonobe-unit.html

en.wikipedia.org/wiki/Sonobe

PAGAT.COM: Pagat is a wonderful collection of card game rules and variations from around the world.

pagat.com

PASCAL'S TRIANGLE: Lessons and links for all grade levels. See also, "All You Ever Wanted to Know about Pascal's Triangle."

mathforum.org/workshops/usi/pascal

ptri1.tripod.com

PROBLEM SOLVING ISLAND: A variety of puzzles, from the book *Thinking Mathematically* and other sources, plus problem-solving tips and sample student journal entries.

math.grin.edu/~rebelsky/ProblemSolving/index.html

QUARTO: A strategy game to play online. Can you get four pieces in a row?

quarto.mygamesonline.org/en

RECREATIONAL MATHEMATICS AT WOLFRAM MATHWORLD: Games, art, humor, and more.

mathworld.wolfram.com/topics/RecreationalMathematics.html

RUSH HOUR ONLINE: "Your goal is to drive your red car out of the playing grid and escape to freedom."

puzzles.com/products/RushHour/RHfromMarkRiedel/Jam.html

SCRATCH: A programming language developed at MIT that makes it easy for students to create interactive stories, animations, games, music, and art—and share those creations on the web.

scratch.mit.edu

SET DAILY PUZZLE: A visual logic puzzle for all ages.

setgame.com/set/puzzle_frame.htm

TIM'S INTERACTIVE PUZZLE SOLUTION CENTER: A collection of "famous and other curious brain teasers," some relatively easy and some quite challenging.

sakharov.net/puzzle

UNIVERSCALE: Compare and understand the relative size of the full range of known objects in our universe.

nikon.com/about/feelnikon/universcale

VISUAL PATTERNS: Fawn Nguyen's algebraic reasoning puzzles. Pick any design and practice recognizing, describing, and predicting the pattern.

visualpatterns.org

WHICH ONE DOESN'T BELONG? PUZZLERS: Mary Bourassa's thought-provoking puzzles will challenge math teachers and students alike. Can you identify which item doesn't belong and explain why?

wodb.ca/index.html

WOULD YOU RATHER? MATH: John Stevens's decision-making challenge. "Asking students to choose their own path and justify it."

wyrmath.com

초등학생과 중학생용

AMBLEWEB FUNCTION MACHINE: Choose the type of problem you want to guess, or go random for more challenge. My math club kids love function machines.

amblesideprimary.com/ambleweb/mentalmaths/functionmachines.html

BEDTIME MATH: Laura Overdeck's daily math problem (with answers) at three levels of difficulty, approximately preschool to upper-elementary level.

bedtimemathproblem.org

DONALD DUCK IN MATHMAGIC LAND: The classic math cartoon from Walt Disney, now free on YouTube.

youtu.be/U_ZHsk0-eF0

DRAGONBOX APPS: Playful math apps that use puzzles to teach the basic principles of numbers, algebra, and geometry.

dragonbox.com

EGYPTIAN MATH: Mark Millmore introduces students to Egyptian numerals and hieroglyph mathematics.

eyelid.co.uk/numbers.htm

ERATOSTHENES' SIEVE: Click on any number, and all of its multiples (except the number itself) will disappear from the chart.

hbmeyer.de/eratostiv.htm

FIBONACCI NUMBERS AND NATURE: Find out all about his famous rabbits, and about honeybees, seashells, pinecones, and more from Ron Knott.

maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html

GRAPH MOLE AND LINE GEM: Two coordinate graphing games by Sulan Dun

funbasedlearning.com/algebra

GRAPHING STORIES: Dan Meyer's fifteen-second videos with action for students to graph. How does the height, air pressure, etc. change with time?

graphingstories.com

HANDS-ON EQUATIONS: These apps take a step-by-step approach to solving simple (linear) algebra equations.

borenson.com/tabid/1594/Default.aspx

HEAD HUNTERS GAME: A bloody fun game for the Viking in all of us. If you enjoy that one, try the other math tricks and games at Murderous Maths.

murderousmaths.co.uk/games/headhunt/headhunt.htm
murderousmaths.co.uk

LOGIC-GRID BRAIN TEASERS: I love logic grid problems, and Braingle offers thirty-four pages of them. See also: Math Brain Teasers.

braingle.com/Logic-Grid.html

braingle.com/Math.html

MATH CATS: Math explorations, crafts, homemade manipulatives, and cats.

mathcats.com

MATH PLAYGROUND: Colleen King shares a variety of math games that make students think—not just number drill.

mathplayground.com

MATH WIRE: Activities and games for elementary students by Terry Kawas. Check out the Hundred Chart Logic Puzzles.

mathwire.com

mathwire.com/problemsolving/hblogic.html

MEGAPENNY PROJECT: How many does it take to pile up a ton of pennies?

kokogiak.com/megapenny

MULTIPLICATION, AN ADVENTURE IN NUMBER SENSE: Explore the multiplication table and discover some interesting things about how numbers work.

web.archive.org/web/20070107073522/http://naturalmath.com/mult/mult1.html

MULTIPLICATION MODELS: How many different ways can you think of to look at multiplication?

web.archive.org/web/20150315214853/http://naturalmath.com/multmodels/index.php

[multmodels/index.php](http://web.archive.org/web/20150315214853/http://naturalmath.com/multmodels/index.php)

ONE IS ONE ... OR IS IT?: One bag of apples, one apple, one slice of apple—which of these is one unit? Video by Christopher Danielson.

youtu.be/EtclWGG7WQ

SUZANNE'S MATH LESSONS: Activities and projects for upper-elementary and middle school, collected by Suzanne Alejandre.

mathforum.org/alejandre/index.html

TAXICAB TREASURE HUNT: A game based on the non-Euclidean geometry of city streets.

learner.org/teacherslab/math/geometry/shape/taxicab/index.html

THINKING BLOCKS: Learn to solve word problems by modeling them with interactive virtual blocks. A visual approach to thinking things through.

thinkingblocks.com

THINKQUEST HISTORY OF MATHEMATICS: Brief overview of math history, with biographies of influential mathematicians and short online quizzes.

web.archive.org/web/20070315145728/http://library.thinkquest.org/4116/

[History/history.htm](http://web.archive.org/web/20070315145728/http://library.thinkquest.org/4116/History/history.htm)

WHO WAS FIBONACCI: “A brief biographical sketch of Fibonacci, his life, times and mathematical achievements” by Ron Knott.

maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibBio.html

WUZZIT TROUBLE: A challenging app that combines an adventure game with mathematical thinking skills, produced by Stanford mathematician Keith Devlin and friends.

wuzzit-trouble.com

연습문제집, 수업시간, 전체 커리큘럼

ACCESS MATHS: A collection of math review worksheets and games.

accessmaths.co.uk

ART OF PROBLEM SOLVING VIDEO LESSONS: For middle school (prealgebra) and up, featuring the ever-entertaining Richard Rusczyk.

artofproblemsolving.com/videos

COOL MATH: Math lessons for prealgebra, algebra, and precalculus, plus math games and other activities.

coolmath.com

CORBETTMATHS 5-A-DAY: Review elementary and middle school math skills with daily practice problem sets.

corbettmaths.com/5-a-day

DAD'S WORKSHEETS: More than 8,000 worksheets you can print for elementary and middle school math practice.

dadsworksheets.com

DONNA YOUNG'S MATH PAGES: Worksheets, charts, drill pages, fraction manipulatives, triangular flashcards, and more.

donnayoung.org/math

FRAYER MODEL PRINTABLE: A graphic organizer designed to provide for a thorough understanding of new vocabulary words.

worksheetworks.com/miscellanea/graphic-organizers/frayer.html

FREE MATH WORKSHEETS FROM HOME SCHOOL MATH.NET: A variety of worksheets from Maria Miller, author of the *Math Mammoth* books, with links at the bottom of the page for more freebies.

homeschoolmath.net/worksheets

FREE PRINTABLES FROM TABLETOP ACADEMY PRESS: Hundred charts, game boards, and graph paper to accompany my *Math You Can Play* series.

tabletopacademy.net/playful-math-books/free-printables

INCOMPETECH: Free online graph paper PDFs galore for any math or science project.

incompetech.com/graphpaper

JUMP MATH: A step-by-step math curriculum for grades 1–8, great for discouraged students or late bloomers. Register for a free account to download the goodies.

jumpmath.org

KHAN ACADEMY: Free video lectures and online quizzes.

khanacademy.org

LIVING MATH HISTORY PLANS: Julie Brennan's curriculum provides a multi-level

structure for home educating or afterschooling families to study math through history and related math topics in context.

livingmath.net/LessonPlans/LessonPlanInformation/tabid/1002/Default.aspx

MATH AS A SECOND LANGUAGE: Herb Gross teaches elementary arithmetic by building on students' intuition about spoken language. Companion videos for teachers at YouTube.

lovemath.org/index.html

youtu.be/QFFReY6lS68?list=PLWwbzNbWxQjvu7zDZbLytz9iUXI8IUN3D

MATH CACHING: Students solve mathematical problems to find hidden "boxes" on the Internet. Each box reveals clues to the location of the next one. Levels range from prealgebra to trigonometry.

mathbits.com/caching/MathCacheDirectionsOpen.html

THE MATH PAGE: Lawrence Spector's interactive lessons in arithmetic, algebra, and assorted other topics.

tbemathpage.com/index.html

MATH WORKSHEET SITE: Scott Bryce's online generator for basic math worksheets, including 1–100 or 0–99 charts. They also offer a subscription service with a wider choice of topics.

themathworksheetsite.com

MATHEMATICS ENHANCEMENT PROGRAMME (MEP): A full curriculum for kindergarten through high school, with so many resources that it's easy to get lost. If you have questions, ask at the MEP Homeschoolers Forum on Yahoo.

cimt.plymouth.ac.uk/projects/mep/default.htm

groups.yahoo.com/groups/mep-homeschoolers

NRICH CURRICULUM MAPS FOR PRIMARY (STAGES 1–2, GRADES 1–5) AND SECONDARY (STAGES 3–5, GRADES 6–12): Resources which will help you embed problem solving into your curriculum.

nrich.maths.org/8935

nrich.maths.org/8517

PROBLEM BASED CURRICULUM MAPS: Geoff Krall has collected many free math activities and lessons, sorted by grade level and arranged to flow in a natural progression.

emergentmath.com/my-problem-based-curriculum-maps

SUCH A THING AS FREE: Could you put together a full year's worth of math from free resources on the Internet? John Golden offers a wealth of links.

mathhombre.blogspot.com/2014/10/such-thing-as-free.html

USING THE REKENREK AS A VISUAL MODEL FOR STRATEGIC REASONING IN MATHEMATICS (PDF): Number sense lessons for the rekenrek by Barbara Blanke. Check out the number rack browser app, too.

bridges1.mathlearningcenter.org/media/Rekenrek_0308.pdf

mathlearningcenter.org/web-apps/number-rack/

대수학과 그 이상의 것

ACTIVE CALCULUS: A free, open-source calculus textbook with activities. See also Screencasts for Active Calculus.

scholarworks.gvsu.edu/books/10

opencalculus.wordpress.com/2015/08/25/screencasts-for-active-calculus

ADJECTIVENOUNMATH.COM: Herb Gross's high school site, with courses on arithmetic, algebra, and calculus. Many of the videos are old-fashioned and slow, but the teacher builds on student intuition to promote understanding better than many of the flashy new sites I've seen.

adjectivenounmath.com/index.html

ALCUMUS: Art of Problem Solving's innovative online learning system adjusts to student performance to deliver appropriate problems and lessons.

artofproblemsolving.com/alcumus

BETTER EXPLAINED: Kalid Azad's intuitive, often visual, explanations of high school math topics.

betterexplained.com

BAYESIAN STATISTICS FOR DUMMIES: Kevin Boone's introduction to statistical questions that deal with conditional probabilities. See also the *New York Times* article "The Odds, Continually Updated."

kevinboone.net/bayes.html

nytimes.com/2014/09/30/science/the-odds-continually-updated.html

CALCULUS CONCEPTS AND APPLICATIONS: A complete set of student-centered activities for a year-long Calculus I and Calculus II sequence.

ibcalculus.com/home

DESMOS GRAPHING CALCULATOR: Explore the relationships between equations and shapes, and try your hands at some of the Daily Desmos blog challenges.

desmos.com

dailydesmos.com

DISCOVERING THE ART OF MATHEMATICS: Activity-based math from a liberal-arts approach.

artofmathematics.org/books

DISCOVERING TRIGONOMETRY: David Eisenberg's basic introduction to trigonometry, starting with sticks and shadows.

catcode.com/trig/index.html

EUCLID'S ELEMENTS: David E. Joyce brings the text of Euclid's thirteen books to life with Java applets. Also see An Introduction to the Works of Euclid.

aleph0.clarku.edu/~djoyce/java/elements/toc.html

obkb.com/dcljr/euclid.html

EUCLID IN COLORFUL DIAGRAMS: Oliver Byrne's innovative, pictorial version of the first six books of the Elements of Euclid.

math.ubc.ca/~cass/Euclid/byrne.html

- FLATLAND: Edwin Abbott's "Romance of Many Dimensions."
ibiblio.org/eldritch/ea/FL.HTM
- G'DAY MATH COURSES: James Tanton unlocks the simplicity of seemingly complex ideas about arithmetic, algebra, probability, and more.
gdaymath.com/courses
- GALLERY OF DATA VISUALIZATION: Michael Friendly's collection of the world's best and worst statistical graphs.
datavis.ca/gallery/index.php
- GEOGEBRA: Download software for playing with geometry and algebra, and the website offers a wealth of user-created instructional materials.
geogebra.org/cms
- GREEN TEA PRESS: Free statistics and computer science books by Allen Downey.
greenteapress.com/wp
- HOW TO READ MATHEMATICS: "A math article usually tells only a small piece of a much larger and longer story." Learn to read between the lines.
web.stonehill.edu/compsci/History_Math/math-read.htm
- HOW TO THINK LIKE A SCHOOL MATH GENIUS: James Tanton's series of videos about five key principles for mathematical thinking for students approaching high school math.
jamestanton.com/?p=1097
- INTERACTIVE MATHEMATICS: Murray Bourne's interactive apps let you explore math and get a better understanding of what it all means.
intmath.com
- INTRODUCTION TO MATHEMATICAL THINKING: Keith Devlin's free online course on language and logic, reasoning and proof.
coursera.org/course/maththink
- INVENT YOUR OWN COMPUTER GAMES WITH PYTHON: In each chapter, Al Sweigart gives your student the complete source code for a new game, then teaches programming concepts from the example.
inventwithpython.com
- JULIA ROBINSON MATH FESTIVAL: Question sets from a collaborative, non-competitive celebration of thought-provoking problems.
jrmf.org/problems.php
- KARL'S CALCULUS: Karl Hahn has a creative way of explaining the ideas of calculus. Includes sample problems.
web.archive.org/web/20140517212513/http://karlscalculus.org
- LA HABRA HIGH SCHOOL'S MATH HISTORY TIMELINE: Math discoveries, publications, and other tidbits from paleolithic number bones to the present.
web.archive.org/web/20080509062108/http://lababra.seniorhigh.net/pages/teachers/pages/math/timeline/MpreAndAncient.html
- LET'S PLAY ANCIENT GREEK GEOMETRY: Nico Disseldorp's online game of complet-

ing Euclidean geometry construction puzzles in a certain number of moves.
sciencevsmagic.net/geo

MAC TUTOR HISTORY OF MATHEMATICS ARCHIVE: My favorite place to begin any foray into math history.

www-history.mcs.st-and.ac.uk

MATHEMATICAL PROBLEMS OF DAVID HILBERT: With a link to Hilbert's 1900 address to the International Congress of Mathematicians in Paris, surely the most influential speech ever given about mathematics. Wolfram MathWorld has an annotated list of all twenty-three problems.

aleph0.clarku.edu/~djoyce/hilbert
mathworld.wolfram.com/HilbertsProblems.html

MATHEMATICAL REASONING: Writing and Proof: Ted Sundstrom's online textbook for high school students and adults who want to develop the ability to think more abstractly.

scholarworks.gvsu.edu/books/9

MEDIAN BY DON STEWARD: A collection of activities and exercises for middle school and high school students.

donsteward.blogspot.co.uk

MUSLIM RULE AND COMPASS: The Magic of Islamic Geometric Design: An article by Alex Bellos, with instructions for constructing a beautiful geometric pattern.

theguardian.com/science/alexs-adventures-in-numberland/2015/feb/10/muslim-rule-and-compass-the-magic-of-islamic-geometric-design

A NEW ALGEBRA: Henri Picciotto offers a selection of interesting activities for algebra students.

mathedpage.org/new-algebra/new-algebra.html

NON-EUCLID: Joel Castellanos's website for exploring the mind-blowingly weird world of hyperbolic geometry. See also the hyperbolic art of M. C. Escher.

cs.unm.edu/~joel/NonEuclid/NonEuclid.html
josleys.com/show_gallery.php?galid=325

PHILLIPS EXETER ACADEMY MATH: An entire high school curriculum made up only of problems.

exeter.edu/academics/72_6539.aspx

PROJECT EULER: A list of programming challenges for advanced students.

projecteuler.net/index.php?section=problems

PROOFS WITHOUT WORDS: I love these.

usamts.org/Gallery/G_Gallery.php

PURPLEMATH: Elizabeth Stapel's explanations for prealgebra and algebra topics. When your textbook just doesn't make sense, look here for help.

purplemath.com/modules/index.htm

SPURIOUS CORRELATIONS: Tyler Vigen takes statistical humor to new heights.

tylervigen.com

STATISTICS EVERY WRITER SHOULD KNOW: Robert Niles's introductory statistics tutorial for math-phobic journalists.

robertniles.com/stats

STATS WITHOUT TEARS: Stan Brown's free online statistics textbook.

brownmath.com/swt

STELLA'S STUNNERS: More than 600 non-routine mathematics problems ranging from simple visual puzzles requiring no specific mathematical background, to problems that use the content of prealgebra up through precalculus.

ohiorc.org/for/math/stella

TAXICAB GEOMETRY INVESTIGATION (PDF): A 19-page printable guiding students as they investigate and make their own discoveries about taxicab geometry. For more advanced taxicab math, see TaxicabGeometry.net.

faculty.cord.edu/andersod/TaxicabWorksheets.pdf

taxicabgeometry.net

THINKING MATHEMATICS VIDEOS: James Tanton's common-sense approach to high school mathematics. See also his Puzzles and Cool Math page.

jamestanton.com/?cat=6

jamestanton.com/?cat=4

TIPS FOR ALL YOUR MATH COURSES: Articles by Stan Brown about how to succeed as a math student, how to use a graphing calculator, what it means to "show your work," and other topics from algebra, trig, calculus, and statistics.

brownmath.com/stfa

TRIG WITHOUT TEARS: "Or, How to Remember Trigonometric Identities." How to learn and understand trig without memorizing a gazillion identities. Check out the author's other mathematics articles, too.

brownmath.com/twt

VIRTUAL MATH LAB: Algebra tutorials from West Texas A&M University. Includes practice tests.

wtamu.edu/academic/anns/mps/math/mathlab

WHAT'S SPECIAL ABOUT THIS NUMBER?: Erich Friedman serves up distinctive facts about several numbers, 0–9999 the last time I looked.

www2.stetson.edu/~efriedma/numbers.html

수학 경연대회

Math contests can be a great source of thought-provoking problems, even for students who dislike competition. Most of these require a registration fee, and most do not accept individual registrations. Organize teams through your school or homeschool group.

MATH KANGAROO: International contest for all grade levels. Students compete individually at their own schools, and then scores are compared nationally. Awards given at school and national levels.

aksf.org

MATHEMATICAL OLYMPIADS FOR ELEMENTARY AND MIDDLE SCHOOLS: International elementary (grades 4–6) and middle school (grades 6–8) levels. Five monthly contests during the school year. Students compete individually in their own schools or homeschool groups—with certificates for everyone and a top-scorer trophy—and then scores are compared nationally for additional awards.

moems.org/index.htm

THE MATH LEAGUE: International contest for fourth grade through high school levels. Students compete individually at their own schools, and then scores are compared nationally. Students may take the test on paper or online.

mathleague.com

MATHCOUNTS: U.S. middle school teams (grades 6–8). Students compete individually at the school level. Each school or homeschool group may send a team of four students to a regional competition, with the top teams progressing to state and national contests (travel required). Study resources available through the MathCounts website and at the following links:

mathcounts.org

- ◆ **MATHCOUNTS DRILLS BY ELIAS SAAB:** Tough online practice problems for MathCounts preparation, or simply to see if you can handle the challenge.

mathcounts.saab.org/mc.cgi

- ◆ **MATHCOUNTS TOOLBOX (PDF):** This is a nine-page summary of the basic facts of elementary math. Go through each page, checking off all the things you know. Then try to learn at least one new math fact per week.

stjudefw.org/mathclub/pdf/coachkit-toolbox.pdf

- ◆ **THE MATHCOUNTS BIBLE ACCORDING TO MR. DIAZ:** “What you must memorize, without excuses and for the rest of your lives (not just for Math Counts).”

unidata.ucar.edu/staff/russ/mathcounts/diaz.html

PURPLE COMET MATH MEET: International contest for middle school and high school teams. Students work together to solve a set of problems and enter their answers online. Mixed-age teams may choose a non-competitive category. Past-years’ contests are available on the website for practice.

purplecomet.org

AMERICAN MATHEMATICS COMPETITIONS: U.S. middle school through high school students compete individually at their own schools, and then scores are compared nationally. Awards given at school and national levels. Past tests available through the website for practice.

amc.maa.org

MANDLEBROT COMPETITION: International high school students compete individually and in teams of four. Ribbons given to top four scorers at the school level, additional awards at the national level.

mandelbrot.org

USA MATHEMATICAL TALENT SEARCH: U.S. high school, or advanced middle school individuals—no team required. Free. Students must solve challenging problems and write well-justified solutions. Four rounds per year, five problems per round,

with one month to work each set. Past problem sets available on the website.
usamts.org

도움을 요청할 수 있는 포럼

ART OF PROBLEM SOLVING FORUM: For middle school and older students, focusing on the puzzling challenge problems found in math competitions.

artofproblemsolving.com/community

ASK DR. MATH: Browse the archives of the Math Forum. If you can't find what you are looking for, ask a new question by email.

mathforum.org/dr.math

FREE MATH HELP FORUM: Help and advice on topics from arithmetic to differential equations.

freemathhelp.com/forum/forum.php

MATH HELP FORUM: Help for algebra and beyond.

mathhelpforum.com/math-help-forum.php

PURPLEMATH FORUMS: "Helping students gain understanding and self-confidence in algebra" and plenty of other high school math topics.

purplemath.com/learning/index.php

S.O.S. CYBERBOARD: A topical help forum for high school and college-level mathematics, engineering, computer science, and more.

sosmath.com/CBB/index.php

부록 C

인용문 및 참조 링크

나는 인용문을 좋아합니다. 내가 인용한 모든 것은 누군가 다른 사람에게 의해 이야기가 전해져 내려왔습니다. 그 사람들 중 적어도 몇몇은 언어로 멋진 삶을 살았습니다.

이 책의 일부 인용문은 내가 책읽기를 통해 찾은 것입니다. 다른 것들은 내가 자주 방문하는 Furman 대학교의 수학과(math.furman.edu/~mwoodard/mquot.html)와 West-field 주립 대학교 수학교육과(wsc.mass.edu/math/faculty/fleron/quotes/)의 웹사이트에서 수집한 것입니다.

AL-KHWĀRIZMĪ, MUHAMMAD IBN MŪSĀ. *The Algebra of Muhammad ben Musa*, edited and translated by Frederic Rosen, Oriental Translation Fund, 1831; available at Google Books.

books.google.com/books?id=3bNDAAAIAAJ

ANDERSON, DOUGLAS R. "Taxicab Geometry Worksheet," Math 105, Spring 2010, Concordia College Moorhead website.

cord.edu/faculty/andersod/TaxicabWorksheets.pdf

BALL, DEBORAH LOEWENBERG. "Magical Hopes: Manipulatives and the Reform of Math Education," *American Educator*, vol. 16 (1992), no. 2, 14–18 and 46–47.

BEECHICK, RUTH. "If you stay with meaningful mental arithmetic ..." from *An Easy Start in Arithmetic (Grades K–3)*, Arrow Press, 1986.

—. "I'll let you in on a secret ..." from *You Can Teach Your Child Successfully*, Arrow Press, 1988.

BELL, E. T. "Compared to what glorious Greece ..." from *Men of Mathematics*, Simon and Schuster, 1937.

BENEZET, LOUIS P. "The Teaching of Arithmetic: The Story of an Experiment," originally published in the *Journal of the National Education Association*, vol. 24–25 (1935–1936); available at the Benezet Centre website.

inference.phy.cam.ac.uk/sanjoy/benezet/three.html

BHĀSKARA ĀCHĀRYA. *Līlāvatī: or, A treatise on arithmetic and geometry*, translated by John Taylor, Courier Press, 1816; available at Google Books.

books.google.com/books?id=OKMIAAAQAAJ

BLANKE, BARBARA. *Using the Rekenrek as a Visual Model for Strategic Reasoning in*

Mathematics, The Math Learning Center, 2008.
bridges1.mathlearningcenter.org/media/Rekenrek_0308.pdf

BOALER, JO. “There is a huge elephant ...” from “Unlocking Children’s Math Potential: 5 Research Results to Transform Math Learning,” academic paper, YouCubed website, 2015. Boaler is a Stanford University Professor of Mathematics Education and the co-founder of YouCubed.org, which hosts a variety of resources to encourage students and parents who struggle with math.
youcubed.org/wp-content/uploads/2015/03/teacher-article-youcubed2.pdf
youcubed.org

BRENNAN, JULIE. “Early exposure to real mathematics ...” from the introduction to the Living Math website. Brennan is a veteran home educator, founder of the Living Math Forum, and the author of the *Living Math Through History* lesson plans.
livingmath.net
groups.yahoo.com/neo/groups/LivingMathForum/info

CARDONE, TINA. *Nix the Tricks: A guide to avoiding shortcuts that cut out math concept development*, self-published, 2013.
nixthetricks.com

CARROLL, LEWIS. “Can you do addition ...” from *Through the Looking Glass, and What Alice Found There*, (Macmillan, 1872); available at Internet Archive. Carroll was the pen name of mathematician Charles Lutwidge Dodgson, who published several books of mathematical puzzles and games, which he felt were valuable as teaching aids, as well as writing serious mathematical papers and textbooks.
archive.org/details/throughlooking00carr

CARSON, RACHEL. “If a child is to keep ...” from *The Sense of Wonder*, Harper & Row, 1965.

DANCIS, JEROME. “What must we add to 4 to obtain 9 as the sum?” from “Reading Instruction for Arithmetic Word Problems: If Johnny can’t read and follow directions, then he can’t do math,” an expansion of his article “When It Comes To Math, Words Count”, *Washington Post*, September 8, 2002, B04; University of Maryland website, July 17, 2007.
www2.math.umd.edu/~jnd/subhome/Reading_Instruction.htm

—. “Supposedly Difficult Arithmetic Word Problems: Keep It Simple for Students,” University of Maryland website, January 2003.
www2.math.umd.edu/~jnd/Difficult_Word_Problems.html

DANIELSON, CHRISTOPHER. “I have spent the last four days ...” from “Let the children play,” Talking Math with Your Kids blog, August 31, 2015. With his blogs and other projects, Danielson helps parents and teachers understand how children understand math.
talkingmathwithkids.com/2015/08/31/let-the-children-play
talkingmathwithkids.com/mathonastick
christopherdanielson.wordpress.com

—. “If you can read with your kids ...” from *Talking Math with Your Kids*, self-pub-

lished, 2013.

talkingmathwithkids.com

- . *Common Core Math for Parents for Dummies*, John Wiley & Sons, 2015. Check out the video resource center with bonus material explaining the concepts and techniques.

dummies.com/Section/id-824962.html

DEVLIN, KEITH. “Mathematical thinking is more than ...” from “What is mathematical thinking?” Devlin’s Angle blog, September 1, 2012. Devlin is a mathematician at Stanford University and the author of many books and several blogs.

devlinsangle.blogspot.com/2012/08/what-is-mathematical-thinking.html

- . *Mathematics: The Science of Patterns: The Search for Order in Life, Mind and the Universe*, Macmillan, 1996.

- . “It Ain’t No Repeated Addition,” Devlin’s Angle, June 2008.

maa.org/external_archive/devlin/devlin_06_08.html

- . “It’s Still Not Repeated Addition,” Devlin’s Angle, July-August 2008.

maa.org/external_archive/devlin/devlin_0708_08.html

- . “Multiplication and Those Pesky British Spellings,” Devlin’s Angle, September 2008.

maa.org/external_archive/devlin/devlin_09_08.html

- . “What Exactly is Multiplication?” Devlin’s Angle, January 2011.

maa.org/external_archive/devlin/devlin_01_11.html

DIRAC, PAUL. “The mathematician plays ...” from a lecture delivered on the presentation of the James Scott prize, February 6, 1939. Published in *Proceedings of the Royal Society (Edinburgh)*, Vol. 59, 1938-39.

damp.cam.ac.uk/events/strings02/dirac/speech.html

DODD, SANDRA. “Unschoolers and Mathematics,” Sandra Dodd website.

sandradodd.com/math

DROUJKOVA, MARIA. “When a kid is feeling bad ...” from a Natural Math Forum discussion of math club activities, February 3, 2011. Droujkova founded Natural Math, an online community to share beautiful, playful, and intuitive mathematics. She is a co-author of *Moebius Noodles: Adventurous Math for the Playground Crowd*.

groups.google.com/d/msg/naturalmath/yW5Pdr8_WhI/b9xULrj1-4wJ

naturalmath.com

EINSTEIN, ALBERT. “It is in fact nothing short ...” quoted by H. Eves in *Return to Mathematical Circles*, PWS-Kent Pub. Co., 1988.

- . “Learning is experience ...” quoted hundreds of times in books and on websites, but I can’t find any reference that credits the original source. Although Einstein may not have said it himself, the quote clearly resonates with many people’s experience of learning.

ERDŐS, PAUL. “Why are numbers beautiful? ...” quoted by Keith Devlin in *The Math Gene: How Mathematical Thinking Evolved And Why Numbers Are Like Gossip*, Basic Books, 2000.

- EVES, HOWARD. “It is impossible to overstate ...” and “There is a distinction ...” quoted by Rosemary Schmalz in *Out of the Mouths of Mathematicians: A Quotation Book for Philomaths*, Mathematical Association of America, 1993.
- FINKEL, DAN. “I’ve spent the last two ...” from “Good Mistakes, Constant Mistakes,” Math for Love blog, February 28, 2009. Finkel runs workshops on mathematics education and is a regular contributor to *The New York Times* Numberplay blog. mathforlove.com/2009/02/82223587
- FLANDERS, J. R. “How much of the content in mathematics textbooks is new?” (1987), *Arithmetic Teacher*, vol. 35 (1987), no. 1, 18–23.
- FURUTA, BURT. “The root problem ...” from “Guest Post: Understanding is Misunderstood,” Math Mama Writes blog, June 12, 2013. mathmamawrites.blogspot.com/2013/06/guest-post-understanding-is.html
- GARDNER, MARTIN. “Biographical history, as taught ...” quoted by George F. Simmons in *Calculus Gems: Brief Lives and Memorable Mathematics*, McGraw-Hill, 1992. Gardner may be best known for his long-standing “Mathematical Games” column in *Scientific American*, but he also wrote more than two dozen challenging books of puzzles and brainteasers.
- . “The best way ...” from *Mathematical Carnival: From Penny Puzzles, Card Shuffles and Tricks of Lightning Calculators to Roller Coaster Rides into the Fourth Dimension*, Random House, 1975.
- . “With a little guidance ...” in the foreword to *Mathematics: A Human Endeavor* by Harold R. Jacobs, W. H. Freeman, 1982.
- GARLIKOV, RICK. “Most adults, including teachers ...” from “‘Explaining’ Math Poorly,” Garlikov website. Garlikov teaches philosophy at Troy University and has written many online articles on philosophy and education. garlikov.com/teaching/badexplan.htm
garlikov.com/schools.htm
- GASKINS, DENISE. “Subtracting Mixed Numbers: A Cry for Help,” Let’s Play Math blog, March 26, 2008. denisegaskins.com/2008/03/26/subtracting-mixed-numbers-a-cry-for-help
- . “If It Ain’t Repeated Addition, What Is It?” Let’s Play Math, July 1, 2008. denisegaskins.com/2008/07/01/if-it-aint-repeated-addition
- . “What’s Wrong with ‘Repeated Addition?’” Let’s Play Math, July 28, 2008. denisegaskins.com/2008/07/28/whats-wrong-with-repeated-addition
- . “Word Problems from Literature,” Let’s Play Math blog, April 26, 2010. denisegaskins.com/2010/04/26/word-problems-from-literature
- . “PUFM 1.5 Multiplication, Part 1,” Let’s Play Math blog, July 16, 2012. denisegaskins.com/2012/07/16/pufm-1-5-multiplication-part-1
- . “Infinite Cake: Don Cohen’s Infinite Series for Kids,” Let’s Play Math, July 2, 2015. denisegaskins.com/2015/07/02/infinite-cake-don-cohens-infinite-series-for-kids
- . “Posts Tagged ‘Math Games,’” Let’s Play Math blog, assorted dates. denisegaskins.com/category/all-about-math/activities/games

- GLAISHER, J. W. L. "I am sure that no subject ..." quoted by Florian Cajori in *A History Of Mathematics*, Macmillan & Company, 1893; available at Google Books.
books.google.com/books?id=bfgRxVzjbMYC
- GLEIZER, OLEG. "Math is freedom ..." in a recorded discussion on the Math 2.0 site, October 20, 2011. Gleizer's Creative Commons (CC BY-NC-SA 3.0) book *Modern Math for Elementary Schoolers* is available free for download.
mathfuture.wikispaces.com/ModernMathOlegGleizer
naturalmath.com/DeltaStreamMedia/OlegGleizerModernMathematics_12_2011.pdf
- GOLDEN, JOHN. "The toughest thing ..." from "Elementary Homeschool," Math Hombre blog, August 2, 2010. Golden's blog includes a variety of games and plenty of sound educational philosophy, and it's well worth browsing.
mathhombre.blogspot.com/2010/08/elementary-homeschool.html
mathhombre.blogspot.com/p/games.html
- . "Instrumental vs. Relational," Math Hombre blog, February 15, 2011.
mathhombre.blogspot.com/2011/02/instrumental-vs-relational.html
- GREENBERG, DANIEL. "And 'Rithmetic" from *Free at Last: The Sudbury Valley School*, Sudbury Valley School Press, 1995; available at Scribd.
scribd.com/doc/14389275/And-Rithmetic-by-Daniel-Greenberg#scribd
- GROSS, HERB. "As important as mathematics is ..." quoted by Jerome Dancis in "Reading Instruction for Arithmetic Word Problems," July 17, 2007. Gross has over 50 years of teaching experience in diverse settings ranging from Central Prison's Death Row in Raleigh, NC, to MIT's Center for Advanced Engineering Study.
www2.math.umd.edu/~jnd/subhome/Reading_Instruction.htm
- . "If we gained weight ..." from "Calculus in Everyday Life," Math as a Second Language website.
adjectivenounmath.com/sitebuildercontent/sitebuilderfiles/calculus_in_everyday_life1.pdf
- . "What's really neat ..." from the Mathematics as a Second Language website.
adjectivenounmath.com/index.html
- GU, WENYUAN. "Were Our Mathematics Textbooks a Mile Wide and an Inch Deep?" academic paper, September 2010, ERIC Archive.
files.eric.ed.gov/fulltext/ED511928.pdf
- HALMOS, PAUL. "Study actively ..." from *I want to be a Mathematician: An Automathography*, Springer, 1985.
- HANKEL, HERMANN. "In most sciences ..." quoted by Stanley Gudder in *A Mathematical Journey*, McGraw-Hill, 1994.
- HARDY, G. H. "A mathematician, like a painter ..." from *A Mathematician's Apology*, Cambridge University Press, 1940.
- HEATH, SIR THOMAS LITTLE. *A History of Greek Mathematics, Volume 1*, Clarendon Press, 1921, available at Google books.
books.google.com/books?id=b4JjsAAAAAAJ

- HOPE, JACK, ET AL. *Mental Math in the Primary Grades; Mental Math in the Middle Grades; and Mental Math in Junior High*, Dale Seymour Publications, 1987–1990. These books appear to be out of print, so try your library or a used book website.
- JACKSON, BILL. “There are 21 girls in a class. There are 3 times as many girls as boys ...” from “The Singapore Math Model-Drawing Approach,” *The Daily Riff*, November 9, 2010.
thedailyriff.com/2010/11/singapore-math-demystified-part-3-the-famous-bar-models.php
- KASNER, EDWARD. “Puzzles in one sense ...” from *Mathematics and the Imagination*, Simon and Schuster, 1940.
- KOVALEVSKAYA, SOFIA. “Many who have never had ...” quoted by W. Dunham in *The Mathematical Universe: An Alphabetical Journey Through the Great Proofs, Problems, and Personalities*, John Wiley & Sons, 1994.
- KOZOL, JONATHAN. “If you could lead ...” quoted in the Mathematical and Educational Quotation Server.
westfield.ma.edu/math/faculty/fleron/quotes/viewquote.asp?letter=k
- KUPPE, MARTIN. “Mathematistan: The Landscape of Mathematics,” from “Mathematics: Measuring \times laziness² (Earthlings 101, Episode 13)” by Zogg from Betelgeuse, YouTube, June 30, 2014.
youtu.be/XqpvBaiJRHo
- LOCKHART, PAUL. “There is no ulterior ...” from “A Mathematician’s Lament,” academic paper, 2002; available at the Mathematical Association of America website. An expanded version was released as a paperback: *A Mathematician’s Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form*, Bellevue Literary Press, 2009. Lockhart teaches math at St. Ann’s School in Brooklyn, New York.
maa.org/external_archive/devlin/LockhartsLament.pdf
- . “I would have expected ...” from the video trailer for his book *Measurement*, Harvard University Press, 2012.
youtu.be/V1gT2f3Fe44
- MASON, CHARLOTTE. “I know you may bring a horse ...” combined excerpts from “Schoolbooks and How They Make for Education,” *The Parents’ Review*, volume 11 (1900), 448–464; and *Towards a Philosophy of Education*, 1923; available at Ambleside Online.
amblesideonline.org/PR/PR11p448Schoolbooks.shtml
amblesideonline.org/CM/toc.html#6
- McMANAMAN, YELENA. “Our own experience ...” from “Math Goggles Challenges,” Moebius Noodles blog, January 21, 2013. McManaman is a co-author of *Moebius Noodles: Adventurous Math for the Playground Crowd*.
moebiusnoodles.com/math-goggles-challenges
- NGUYEN, FAWN. “I’m not worried about ...” from the “Teachers” page at her Math Talks site. Nguyen teaches math at Mesa Union Junior High, blogs about her

students' learning, and moderates the fantastic Visual Patterns website.

mathtalks.net/teachers.html

fawnnguyen.com

visualpatterns.org

PADRON, ROBIN. "If we have never ..." and "Story time is precious ..." from *How to Homeschool Math—Even if you Hate Fractions!!* Self-published, 2011. Robin teaches math in Connecticut and does private math tutoring for public- and private-school students, as well as for homeschoolers.

PAULOS, JOHN ALLEN. "If mathematics education ..." from *Innumeracy: Mathematical Illiteracy and Its Consequences*, Macmillan, 2011.

PESEK, DOLORES D. and David Kirshner. "Interference of Instrumental Instruction in Subsequent Relational Learning," *Journal for Research in Mathematics Education*, vol. 31 (2000), no. 5, 524–540.

academic.sun.ac.za/mathed/174/reinstrumental.pdf

PÉTER, RÓZSA. "I love mathematics ..." quoted by Rosemary Schmalz in *Out of the Mouths of Mathematicians: A Quotation Book for Philomaths*, Mathematical Association of America, 1993.

POINCARÉ, JULES HENRI. "The mathematician does not ..." quoted by H.E.Huntley in *The Divine Proportion: A Study in Mathematical Beauty*, Dover Publications, 1970.

POLIKOFF, MORGAN S. "The Redundancy of Mathematics Instruction in U.S. Elementary and Middle Schools," *The Elementary School Journal*, vol. 113 (2012), no. 2, 230–251.

web-app.usc.edu/web/rossier/publications/66/The%20Redundancy%20of%20Math%20Instruction.pdf

POLYA, GEORGE. "A great discovery ..." and "A teacher of mathematics ..." and "Solving problems..." from *How to Solve It: A New Aspect of Mathematical Method*, Princeton University Press, 1957.

ROTHSTEIN, EDWARD. "Real mathematics is not ..." from "Making Math Magical," *Family Life*, September/October 1996.

RUSSELL, BERTRAND. "It seemed to me ..." from *My Philosophical Development*, Allen and Unwin, 1959.

—. "Mathematics, rightly viewed ..." from "The Study of Mathematics," *Mysticism and Logic: And Other Essays*, Longmans, Green and Company, 1919; available at Google Books.

books.google.com/books?id=zrwMQAAAAYAAJ

SAWYER, W. W. "The best way to learn ..." and "Everyone knows that it is easy ..." from *Mathematician's Delight*, Penguin Books, 1943; available at Internet Archive. Sawyer was a British mathematician and educator whose life spanned most of the twentieth century.

archive.org/details/MathematiciansDelight

www.sawyer.org/default.html

—. "A widespread fallacy ..." and "It is not enough for a child to hear ..." and "Most

- remarks made by children ..." from *Vision in Elementary Mathematics*, Penguin Books, 1964.
- SENGER, NICK. "Paraphrasing is one of ..." from "Quotes about Reading for Students to Ponder and Paraphrase," Teen Literacy Tips blog, September 12, 2007.
nicksenger.com/blog/quotes-about-reading-for-students-to-ponder-and-paraphrase
- SKEMP, RICHARD. "Suppose that a teacher reminds a class ..." from "Relational Understanding and Instrumental Understanding," *Mathematics Teaching*, 77 (1976), 20–26.
grahamtall.co.uk/skemp/pdfs/instrumental-relational.pdf
skemp.org.uk/
- STEEN, LYNN ARTHUR. "As biology is the science of life ..." from "Reflections on Mathematical Patterns, Relationships, and Functions," academic paper prepared for the Minnesota K–12 Mathematics Framework, SciMath-MN, Minnesota Department of Children, Families, and Learning, 1998. See also "The Science of Patterns," *Science*, vol. 240, no. 4852, April 29, 1988.
stolaf.edu/people/steen/Papers/98algebra_mn.pdf
stolaf.edu/people/steen/Papers/88sci_patterns.pdf
- STIGLER, JAMES W. and James Hiebert. *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*, Simon and Schuster, 2009.
- STROGATZ, STEVEN. "The Elements of Math," *The New York Times*, January–May 2010.
topics.nytimes.com/top/opinion/series/steven_strogatz_on_the_elements_of_math/index.html
- . *The Joy of x: A Guided Tour of Math, from One to Infinity*, Eamon Dolan/Houghton Mifflin Harcourt, 2012.
- TANTON, JAMES. "The high-school English curriculum ..." from "Goal of this Site," Thinking Mathematics website. Tanton worked as a college professor for a decade, as a high-school teacher for a decade, and is now the Mathematician in Residence at the Mathematical Association of America in Washington D.C.
jamestanton.com/?page_id=60
- TOOM, ANDRE. "Word problems are very valuable ..." excerpted from "Word Problems in Russia and America," academic paper, November 6, 2010; extended version of a talk at the Meeting of the Swedish Mathematical Society, June, 2005. Check out Toom's other articles on math and education, too.
de.ufpe.br/~toom/travel/sweden05/wp-sweden-new.pdf
toomandre.com/my-articles/engeduc/index.htm
- . "For example, coins ..." from "Word problems: Applications vs. Mental Manipulatives," *For the Learning of Mathematics*, v. 19 (1), March 1999.
toomandre.com/my-articles/engeduc/manipul.pdf
- VANHATTUM, SUE. *Playing with Math: Stories from Math Circles, Homeschoolers, and Passionate Teachers*, Delta Stream Media, 2015. VanHattum is a community college mathematics teacher, math circle leader, and blogger.

playingwithmath.org

WAY, JENNI. "Learning Mathematics Through Games Series," Nrich Enriching Mathematics website.

nrich.maths.org/2489

WHITEHEAD, ALFRED NORTH. "From the very beginning ..." from *The Aims of Education and Other Essays*, Williams & Norgate Limited, 1938.

—. "I will not go so far ..." quoted by W. H. Auden and L. Kronenberger in *The Viking Book of Aphorisms*, Viking Press, 1966.

WHITNEY, HASSLER. "No wonder you hate math ..." quoted by Claudia Zaslavsky in *Fear of Math: How to Get Over It and Get On with Your Life*, Rutgers University Press, 1994.